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CAN POPULATION GROW FOR EVER?

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Even advocates of space colonisation commonly assume that in the long term population growth must cease. This belief is examined and rejected. A strategy is outlined that would permit mankind to grow in population, knowledge and power forever. Certain difficulties that may arise in the far future of the Universe are considered and possible solutions put forward. It is shown that although an open universe does not succumb to the 'heat-death' the decay of very distant matter before mankind can reach it may be a problem.

1. INTRODUCTION

Thomas Malthus (1766-1835) enunciated the doctrine that populations grow geometrically whereas production grows at most linearly, so that the time must come when there is insufficient food, or coal, or steel for all [1-3]. The notion that population growth is inevitable, leading ultimately to disaster – economic collapse, starvation, extinction – we now call Malthusian.

This doctrine carried little weight with Victorian economists, for the predicted catastrophe quite failed to materialise. It was largely forgotten. But in the past few decades there has been a resurgence of concern about global population growth, an increased awareness of the so-called "Third World" and a widespread loss of faith in technology.

The Neo-Malthusian doctrine claims that resources are finite, that reserves are nearly exhausted and that the world population must be controlled at any cost. This goes beyond what Malthus wrote both in its pessimism and its harsh demands, which are now disseminated with a moral fervour not far from fanaticism. Exemplifying this new trend are a number of influential books [4-6] notable more for emotional rhetoric than for rigorous argument.

These works have been so successful as propaganda, however, that their ideas of the "limits to growth," the "dwindling reserves of non-renewable resources," the "population explosion" and the need for "zero population growth" (ZPG) are today accepted by the majority of people in the industrialised world. Their philosophy pervades the educational establishment, literature and the media; and millions of pounds are being spent on discouraging people from having children. Only a few brave voices [7] now resist this doctrine of doom.

Space activists are well aware of the potential of the industrialisation of space to solve many problems on Earth: Solar Power Satellites to provide energy, asteroid mining to provide raw materials, orbital factories to cure pollution, and so on and so on. They see the idea that there is "Only One Earth" [8] as absurd. These issues have been examined at length elsewhere [9-11] and need not be further enlarged upon.

However, even advocates of space colonisation have often absorbed, perhaps unwittingly, elements of the Malthusian ethos. Whilst claiming that "the end is not yet" they still accept the ultimate necessity for ZPG or that resources are finite [12-15]. Space colonisation, it is said, cannot "solve the population problem," since an advanced civilisation would still have to "control" its population. All too often, it is only argued that the use of resources from space can delay their eventual exhaustion.

A fundamental fallacy of the Neo-Malthusian school of thought lies in the claim that "natural resources are finite," for which there is not the slightest evidence (it is usually held to be "obvious"). On the contrary, what historical and scientific evidence we possess suggests that resources are *not* finite in any meaningful way, either physically or operationally. Throughout human history available resources and reserves have grown faster than the population, still do so today and can be expected to continue to do so in the future.

The economic effects of population growth are considered in Ref. 7, which, although considering space colonisation only in passing, gives a clear picture of the interesting and generally positive link between population growth and the availability of resources.

A somewhat different objection to indefinite population growth is that migration would be too costly or too slow, even if enough new territory could be found. In the field of interstellar colonisation Ref. 16 purported to show that exponential population growth must cease, because of the limited speed of migration possible from a growing sphere of influence. This idea has been used by CETI supporters as a rationalisation of the Fermi Paradox ("if extraterrestrials exist, where are they?" [17-18].

In this paper we shall argue that such considerations are misguided, and that there is every reason to believe in the potential for indefinite population growth.

2. LIMITS TO POPULATION GROWTH

In the past much population growth has been linked with technological innovations allowing greater population densities. It has been argued that there is a limit to this process, and this is surely plausible, since human beings as we know them require at least ~100kg of mass and ~100W of power to exist. We shall not be arguing in this paper for an ever-growing population density, but on the contrary will generally consider an ever-increasing mass and power per capita.

(However, it should be pointed out that if consciousness obeys the Biological Scaling Hypothesis of Ref. 19 it may in fact become possible to overcome the population density limit and embody persons with arbitrarily low mass and power requirements).

2.1 Velocity-Limited Population Growth

Consider a process of one-dimensional migration (a primitive tribe spreading along a river valley) where the average velocity of migration cannot exceed a value v and the population per unit length ρ is constant. The population at late times will be:

$$P = \rho vt \quad (1)$$

Initially, when the population is less than given in Eq. (1), a period of exponential growth is possible, such that:

$$P = P_o \exp(\alpha t) \quad (2)$$

which can only continue until:

$$P_o \exp(\alpha t) = \rho vt \quad (3)$$

To put this into perspective, we note that, using typical values of $P_o=2$, $\alpha=0.02yr^{-1}$, $\rho=1m^{-1}$ and $v=0.01ms^{-1}$, then the period of exponential growth is nearly 1000 yr. Such a tribe would by then have grown to $\sim$300 million strong, over some 300,000 km of river valley.

It would appear that historically the maximum velocity of migration has not in itself limited population growth.

2.2 Migration in More Than One Dimension

Clearly migrations have not been restricted to one-dimensional river valleys, but have also crossed plains and seas. For such 2-D migration the long-term velocity-limited growth is quadratic:

$$P = \rho(vt)^2 \quad (4)$$

and exponential growth ends when:

$$P_o \exp(\alpha t)=\rho(vt)^2 \quad (5)$$

Using the same values as in Section 2.1 (but with $\rho=10^{-4}\, m^{-2}$ in 2-D) we obtain a final population $\sim 2 \times 10^{13}$ after 1500 yr of exponential growth, a large increase upon the 1-D case.

In the 3-D colonisation of interstellar space the limiting growth curve is cubic:

$$P = 4\pi/3.\ \rho(vt)^3 \quad (6)$$

and exponential growth ends when:

$$P_o \exp(\alpha t)=4\pi/3.\ \rho(vt)^3 \quad (7)$$

Typical values that have been suggested as appropriate are $P_o \sim 10^{10}$, $v\sim 0.1$ ly/yr and $\rho\sim 10^{13}\, ly^{-3}$ (conservatively assuming a population $\sim 10^{15}$ per solar system), leading to a population $\sim 6 \times 10^{19}$ after about 1100 yr when the sphere of influence is about 110 ly in radius. Even if we put $v=1$ ly/yr and $\rho=10^{20} ly^{-3}$ exponential expansion ends after about 2400 yr and 2400 ly, although only a small fraction of the Galaxy has then been reached.

It would therefore appear that Ref. 16 is correct and that exponential growth must cease before the Galaxy is colonised. But we have argued too quickly and made several unwarrented assumptions.

First, we have assumed that the migration velocity is constant, but historically this velocity has increased with population growth and technological advance, a trend which we expect to continue. If the migration velocity obeys

$$v = v(t)=v_o \exp(\nu t) \quad (8)$$

then a population growth rate of $\alpha=3\nu$ is sustainable indefinitely. It may be objected that v cannot exceed the velocity of light and that Eq. (8) is therefore invalid, but we shall see in Section 3 that, for migration at least, the velocity of light barrier is illusory; this is quite apart from any possibility of FTL travel [20].

Second, we have assumed that the dimensionality is constant (and equal to three), but historically the dimensionality of migration has increased with population growth and technological advance, a trend which we expect to continue. It is speculative, but not unreasonable, to extrapolate beyond a dimensionality of three for the future. This may be possible, even in 3-space, if the topology is suitably complex, and black holes may provide this topological complexity (compare this with the effective dimensionality approaching two of the tributaries and distributaries of a nominally 1-D river valley).

2.3 Continued Exponential Growth in Velocity-Limited Migration

Even in the power term phase of population growth limited by the velocity of migration it remains possible for exponential growth to continue for part of the population.

A simple linear picture, in which each colony founds the next colony in line, is adequate for modelling the radial progression of the colonisation front after the initial phase of exponential growth.

Let the population growth rate in each colony follow:

$$\alpha_i = \alpha(1-P_i/P_{max}.) \quad (9)$$

where P_i is the population of the i-th colony, P_{max} the maximum population per colony and α the free growth rate. Then the population of each colony will grow exponentially initially and approach the maximum value asymptotically (a standard S-curve).

It will be seen that the border colonies are always in the exponential growth phase, where both the growth rates and emigration rates are high. Only behind the front does the growth rate gradually fall, eventually approaching (but never reaching) 'zero population growth.' It is important to note that in such a scenario population 'control' is never necessary for the pioneer families driving the colonisation front, only in the long-settled regions far behind it. In the case of interstellar colonisation each colony could grow exponentially for about 1500 yr. (A similar growth pattern occurred during the frontier days of America.)

It is therefore apparent that, even if migration is velocity-limited, enforced ZPG, with its radical social implications, is unnecessary, improbable and undesirable. The argument that colonisation of the Galaxy will cease as a result of the social change of ZPG, so purportedly resolving the Fermi Paradox [17], is thus seen to be flawed; and the conclusions of the Hart-Viewing chauvinists [21-23], that extraterrestrial intelligent beings do not exist, are upheld. It is because of this that I implicitly assume the uniqueness of mankind; however, if in fact ETI exist the main conclusions of this paper can be readily adapted.

3. STRATEGY FOR INDEFINITE POPULATION GROWTH

In this section we shall derive a relativistic strategy, which is not "velocity limited," for indefinite population growth. It will not make use of hypothetical new physics (such as FTL travel) and will permit net per capita industrial growth, so that wealth will increase along with population.

3.1 Euclidean Space-Time

We begin by considering a static universe of constant energy density in Euclidean (flat) space-time; this will give a good description for colonisation out to about the Hubble radius (~3 Gpc), but thereafter a more sophisticated analysis will be needed.

We assume that the population grows exponentially and encompasses a volume V at a given time t. In order to emigrate to a new world it is necessary to travel, on average, a distance $d \sim V^{1/3}$, in a constant (or at most logarithmically increasing) length of time as experienced by the migrant.

Thus the journey time must be carried out at a Lorentz factor $\gamma \propto d$, such that the elapsed time on board is $\tau \sim d/\gamma c$. This means that the proper speed $\sim \gamma c$ is unlimited.

To the rest of the Universe the migration wave travels at the speed of light (close enough), but subjectively or 'historically' the effective speed is γc and ever-increasing. The energy consumed is proportional to the Lorentz factor and the proper speed.

For a population P, population density ρ and historic time t:

$$P = P_o \exp(\alpha t) \quad (10)$$

$$\rho = \rho_o \exp(-\beta t) \quad (11)$$

$$V = P/\rho = (P_o/\rho_o) \exp((\alpha+\beta)t) \quad (12)$$

$$\gamma \propto d \propto V^{1/3} \propto \exp((\alpha+\beta)t/3) \quad (13)$$

The emigration rate per unit volume is proportional to the population density, so the power density requirement is given by:

$$P \propto \rho \propto \gamma \propto \exp(((\alpha + \beta)/3 - \beta)t) \quad (14)$$

But if the power density is held constant we obtain:

$$\beta = \alpha/2 \quad (15)$$

This strategy will allow continuous population growth with $P \propto \exp(\alpha t)$, $\rho \propto P^{-1/2}$, $V \propto P^{3/2}$ and $\gamma \propto d \propto P^{1/2}$. If we equate industry with power consumption the industrial growth rate is 3/2 times the population growth rate (gross) and 1/2 (net per capital). In economic terms this is not hard to achieve, since typically the industrial growth rate is about twice the population growth rate.

The emigration rate per capita is here $(\alpha+\beta)=3\alpha/2$ and the total emigration per unit volume is $(\alpha+\beta)\rho_o/\beta=3\rho_o$. Thus the material resources required per unit volume over all time remains finite.

Suppose the total available energy per unit volume to be limited, such that the available power density decreases exponentially:

$$p = p_o \exp(-\epsilon t) \quad (16)$$

We may put:

$$(\alpha+\beta)/3-\beta=-\epsilon<O \quad (17)$$

Typically, however $\alpha \sim \beta \sim 10 yr^{-1}$ whereas $\epsilon \leq 10^{-10} yr^{-1}$ for astronomical power sources; hence:

$$\beta=\alpha/2+3\epsilon/2 \simeq \alpha/2 \quad (18)$$

That is, a very slight additional decrease in population density allows exponential growth to continue, even though the total energy density be finite and decreasing exponentially, perhaps through a process such as proton decay [24].

3.2 Open Universe

In an open universe the geometry of space-time is hyperbolic. In the standard models the universe is infinite in extent and will expand forever [25]. There is evidence that our universe is of this type [26].

For colonisation beyond the Hubble radius (redshift z= 1) the expansion of the universe becomes most important. If the cosmic time is ξ the proper distance to a galaxy is [19]:

$$d \sim \psi \exp(\xi) \quad (19)$$

where ψ is a normalised angular distance parameter related to the curvature of space-time. The number of galaxies within this distance scales as [19]:

$$N \sim \exp(2\psi) \quad (20)$$

If the galaxies were mutually at rest it would take a time $d/c \sim \psi \exp(\xi)$ to reach such a galaxy, but because of their mutual recession we find that this time is increased by a factor $\sim \exp(\psi)$. Thus the Lorentz factor required for the migrants' journey:

$$\gamma \sim \psi \exp(\psi) \exp(\xi) \quad (21)$$

The term $\exp(\xi)$ is the time elapsed from the origin of the universe in Hubble periods, which scales as t at late times. We have:

$$\gamma \sim \ln N . N^{1/2} . t \quad (22)$$

Since $N \sim \exp((\alpha+\beta)t)$ for exponential population growth:

$$\gamma \sim t^2 \exp((\alpha+\beta)t/2) \quad (23)$$

The required power density is therefore:

$$p \sim \rho\gamma \sim t^2 \exp((\alpha-\beta)t/2) \quad (24)$$

For constant or slowly decreasing power density, noting that the exponential outweighs any power term, we obtain:

$$\beta \simeq \alpha \quad (25)$$

This strategy allows for continuous growth with $P \propto \exp(\alpha t)$, $\rho \propto P^{-1}$, $N \propto P^2$, $\psi \propto t$ and $\gamma \propto t^2 P$ at late times. The emigration rate per capita is 2α and the total emigration per unit initial volume ρ_o. The total energy required per unit initial volume (at cosmic times ξ_o) $\sim \int t^2 \exp((\alpha-\beta)t/2)dt$ which converges provided that $\beta-\alpha=\epsilon>0$; thus both material and energy requirements per galaxy remain finite over all times. The gross industrial growth rate for this strategy is twice the population growth rate.

3.3 Closed Universe

It may be that the universe is actually closed and finite. The strategy given above would then only work for a limited time, until the whole universe had been filled up.

We may speculate as to how further expansion might then be made possible. For example, General Relativity apparently predicts that traversing the ergosphere of a large rotating black hole may bring one into another universe containing a copy of the black hole [28]; repeated passes through the black hole would give access to an infinite series of universes. It may even be that we shall eventually be able to create universes at will, merely by collecting stars together into a black hole. If something of the sort is true, then the

TABLE 1. Timescales for Astronomical Processes (from Refs. 19 and 27).

Astronomical Process	Astronomical Duration-yrs	Historical Time from Present-yrs
Main sequence star formation and evolution	10^{11}	3300
Formation of stellar black holes	10^{11}	3300
Low mass stars cool down	10^{14}	3700
Planets detached from stars	10^{15}	3800
Stars detached from galaxies	10^{19}	4300
Formation of galactic black holes	$\lesssim 10^{20}$	$\lesssim$ 4400
Decay of orbits by gravitational radiation	10^{20}	4400
Proton decay	10^{31}	5600
Decay of stellar black holes	10^{64}	9400
Matter loses structural strength (cold flow)*	10^{65}	9600
Decay of galactic black holes	10^{97}	13200
Positronium formation and decay	$\gtrsim 10^{116}$	$\gtrsim$15400
Quantum tunneling of nuclei to iron *	10^{1500}	175000
Quantum tunneling of matter into black holes	$10^{10^{26}}$	$1.2\text{x}10^{28}$

* if nucleons are stable (protons do not decay)

effective extent and dimensionality of space-time would be infinite (or, more strictly, indefinite).

We may also note that in an open universe of infinite extent and marked inhomogeneity there may exist large relatively dense regions that appear from within as closed universes [29]; such regions would form "bubbles" or "island universes" in a wider sea. By the same token a closed universe might be locally "burst open" [30] to expand forever or give access to a wider universe that may be truly infinite.

It must be emphasised that these suggestions are highly speculative. Nevertheless, they illustrate the fact that even a closed and finite universe may not place any ultimate limits upon mankind.

4. DIFFICULTIES AT LATE TIMES

The simple scenario given in Section 3 encounters certain difficulties at late times, not all of which appear readily solvable at present. Some, such as the construction of relativistic starships, are straightforward technological problems for which outline solutions can be given. Other are more profound.

4.1 Elapsed Cosmic Time in Distant Galaxies

The time taken for a colonist to reach a distant galaxy may be only decades for him, but for the universe a long time will have elapsed. A galaxy at arc parameter ψ can be reached at cosmic time ξ:

$$\xi \sim \psi \tag{26}$$

Since the arc parameter is also proportional to the historical elapsed time t we have:

$$\xi \sim t(\alpha+\beta)/2 \tag{27}$$

If the arrival time, counting from the beginning of the universe, is $T \sim \exp(\xi)$ we find that typically, at times $t >> 50$yr:

$$T \sim 10^{-18}\text{yr} \,.\, \exp(T/50\text{yr}) \tag{28}$$

Thus after a short span of millenia in historical time the colonists may arrive at their destination in a very much older universe. When $T \sim 10^{12}$yr it is likely that most of the stars will be dead, yet this circumstance may be met by colonists within ~3500 years of history.

If no energy sources survive for the colonists' further use then migration will – apparently – have to cease. Although the stars will go out in a comparatively short time, other energy sources – gravitational potential energy, proton decay, black hole evaporation – will last far longer; Table 1 gives timescales for these phenomena, both in cosmic time and our putative historical time.

Whether any form of matter or usable energy (such as dust grains or positronium) can be expected to survive in appreciable quantities forever naturally (without intelligent intervention) is uncertain, but at present it appears unlikely. This may therefore be a fatal flaw in the expansion scenario at late times, and as yet no definite solution has been found. (However, we may note that FTL travel might provide such a solution – and see also Section 5.4).

4.2 Population within Horizon

In the scenario given above the population density falls with time. If we consider a minimum society of a certain number of persons it is apparent that its physical extent must increase with time. The limit to this process is evidently the amount of matter within the causal horizon which even at late times scales only as t^2. Thus the exponential decrease of population density will in due course lead to an arbitrarily low population (less than unity!) within the horizon. (The concept of a causal horizon does assume that there is no FTL travel).

To overcome this limit it would appear that the population density must decrease no faster than t^{-2} at late times. This is likely to necessitate a comparable reduction in emigration and population growth rates, but at no time would the growth rate actually become zero or negative. Growth remains possible within a growing causal horizon.

4.3 Decay of Matter within Horizon

In order for a society to grow in population and power indefinitely the amount of available mass and energy within the horizon must continue to grow. But at late times exponential decay of matter (proton decay, etc.) would become important, since the mass$\sim t^2 \exp(-\epsilon t)$ would eventually tend to zero, and life (at least on our restrictive assumptions) would become impossible.

To avoid this fate we must find ways to store matter and energy stably, or with an increasing decay time $t_{dec} \sim t^r$, $r \gg 1$.

For black holes the decay time $\sim M^3$, and if we bring together black holes of mass $M \sim t^2$ (a constant fraction of the mass within the horizon) the decay time increases rapidly $\sim t^6$, while the emitted power falls $\sim t^{-4}$. If instead one uses ever more black holes of mass $M \sim t^{1/3}$ then $t_{dec} \sim t$ and the emitted power within the horizon $\sim t$. Between these limits lie a range of possibilities.

4.4 Extraction of Black Hole Energy

Energy may be extracted from black holes either by collecting the power emitted as they decay (a quantum mechanical phenomenon) or by extracting the mutual gravitational energy of coalescing black holes or the rotational energy of a spinning black hole classically.

A black hole of mass M has a temperature $T \sim M^{-1}$, a decay time $t_{dec} \sim M^3$ and an emitted power $P \sim T^4 R^2 \sim M^{-2}$. We can immediately see that if the black hole temperature is not to fall below the background temperature (which scales as t^{-1}) we must have black holes of mass $M \sim t^r$ where $1 \geq r \geq 1/3$.

In order to capture the radiation of the black holes we must surround them with absorbing shells. If we employ a mass M_{cap} for this purpose it must be supported by a pressure $p \sim (M_{cap}/R^2)(M/R^2) \sim M_{cap}/M^3$, which implies a total support energy and mass $\sim M_{cap}$ for $R \sim M$. The amount of capture mass may be estimated in several ways: the number of electrons needed to absorb the black hole radiation is $Ne \sim P/T^3 \sim M$; and a complete shell of dipoles of constant cross-section and length $\sim T^{-1} \sim M$ has a mass $\sim R \sim M$. Thus the total mass required for capturing the radiated power is proportional to the mass of the black hole itself.

However, because of proton decay or other disordering mechanisms the capture mass must be repaired on a fixed timescale and therefore demands a constant specific power. Since $P \sim M^{-2}$, $P/M_{cap} \sim M^{-3}$, so the size of useable black hole is limited (unless a structure requiring repair only on increasing timescales can be found). This is the problem met in Ref. 27.

This problem may be overcome by maintaining two black hole populations; one with mass $M \sim t^{1/3}$ for indefinite storage of mass and energy; another of black holes allowed to decay from the storage mass to a standard mass useable as a power source at constant specific power. But only a decreasing fraction $\sim M^{-1}$ of the stored energy could be used.

We may also generate power by classical (i.e. not quantum mechanical) gravitational interactions. For example, a smaller body orbiting a rotating black hole or a binary pair of black holes will experience accelerations $\sim M/R^2 \sim M^{-1}$ and can extract energy at a specific rate $\sim M^{-1}$ (for a standard velocity $\sim c$). The time taken to extract all the available energy will vary as M^2/M_{sub}, so the subsidiary mass should not be reduced faster than M^{-1}.

If the subsidiary mass were of normal matter it would decay in a fixed time (since the specific power $\sim M^{-1}$), but if it consisted of one or more black holes with $M_{sub} \sim M^{1/3}$ (or greater) there would be enough power for their repair or replacement. The energy extraction would then take a time $\sim M^{5/3} \sim t^{5/9}{}_{dec}$. Since the ultimate extraction to normal matter requires a constant specific power and standard black hole mass, additional stages, utilising smaller and smaller black holes, would be needed. However, the number of stages would grow very slowly $\sim \log\log M$, and although there might be a certain loss at each stage the overall loss would only grow as $\log M$; typically the number of stages would be $\sim(2.1\log(33+\log(M/Mo))-3.5)$.

This multi-stage classical scheme for the extraction of energy from black holes appears to offer the efficiency and flexibility we desire at all epochs, and to overcome the problems caused by proton decay and other disordering mechanisms.

5. SCENARIOS FOR BLACK HOLE GROWTH

If the thermal radiation of black holes is to be used it will be appropriate to put $t_{dec} \sim t$ and $M \sim t^{1/3}$. More generally we put $M \sim t^r$. The total mass used to create a black hole of mass M we shall call M_T.

Black holes must be brought together over increasing distances $\sim t.\ell n M_T$ over a time interval $\sim t$, against the expansion of the universe with redshift $\sim \ell n M_T$. A cumulative specific energy $\sim \gamma \sim (\ell n M)^2$ is thus required.

5.1 Coalescence of Black Holes

When two black holes coalesce the mass of the resulting black hole is greater than the mass of either but less than the sum. If two black holes of mass M coalesce to form one of mass 2Ma, the amount of energy released will be 2M(1-a). According to the laws of Black Hole Thermodynamics [31] $1/2 \leq a \leq 1$ and for typical energy extraction mechanisms $a \simeq 0.8$.

After n stages of coalescence (starting with black holes of mass M_o) the black hole mass is $2^n M_o a^n$ and the cumulative energy released is $2^n M_o(1-a^n)$. The cumulative specific energy $E=(1-a^n)/a^n \sim a^{-n}$.

Given that $M \equiv 2^n M_o a^n \sim t^r$ we have $n \sim \ell n t$; substituting for n yields:

$$E \sim a^{-r \ell n t / \ell n 2a} \sim t^{-r/(1+\log_a 2)} \tag{29}$$

This varies from zero for a=1 (no conversion), through $t^{r/2}$ for $a \simeq 0.8$, to infinity as $a \rightarrow 1/2$ (black hole mass not increasing). The total mass used grows faster than the black hole mass itself as:

$$M_T = 2^n M_o \sim t^r/a^n \sim t^{r/(1+\log_2 a)} \tag{30}$$

This still demands only a logarithmically growing cumulative specific energy, by comparison with a power-law release; there is thus ample energy for black hole collection at all times.

Now if the trapped energy fraction should decrease with mass as $M^{-\mu} \sim t^{-\mu\alpha}$ the cumulative specific energy obtained will be reduced to:

$$E=((1-a)+a(1-a)(2a)^{-\mu}+a^2(1-a)(2a)^{-2\mu} +\ldots)/a^n \tag{31}$$

$$\therefore \quad E=(1-a)/a^n.((a(2a)^{-\mu})^n-1)/(a(2a)^{-\mu}-1) \tag{32}$$

It may be seen that we still have $E \sim a^{-n}$, the actual value being changed only by a factor of order unity.

5.2 Black Holes within Causal Horizon

Since the total mass-energy within the causal horizon scales as t^2 the total number and mass of black holes within the horizon will scale as:

$$N \sim t^{2-r/(1+\log_2 a)} \tag{33}$$

$$NM \sim t^{2+r/(1+\log_a 2)} \tag{34}$$

The total available power within the horizon is:

$$P_T \sim t^{2-1+r/(1+\log_a 2)-\mu r/(1+\log_2 a)} \tag{35}$$

$$\text{i.e. } P_T \sim t^{1+r(1-\mu\log_a 2)/(1+\log_a 2)} \tag{36}$$

The rate of increase of available power is maximised when $r=1/3$ (as its minimum allowed value) and when $\mu=0$ (although the power still increases even if $\mu=1$).

Since a classical multi-stage scheme can provide $\mu \simeq 0$ at $a \simeq 0.8$ and $r=1/3$ we may choose such a scenario yielding $N \sim t^{1.51}$, $NM \sim t^{1.84}$ and $P \sim t^{0.84}$. The sustainable amount of normal matter $\sim P_T \sim t^{0.84}$, so both population and per capita wealth may continue to grow indefinitely.

5.3 Information Storage and Processing

Continued growth without the memory or historical records of times past and an accumulation of scientific knowledge and works of art would be of dubious value, so we must ask whether sufficient data can always be stored and processed.

Within the Earth-like environments information can be handled at a rate $\sim P_T \sim t^{0.84}$. But data can also be stored in the form of zero-rest-mass particles or radiation in space, where the energy per bit can scale as $T \sim t^{-1}$. Since the radiative energy within the horizon $\sim t^2$ the storage capacity within the horizon $\sim t^3$; this compares favourably with what is needed to allow a constant storage bit rate per unit of power consumed, which scales only as $t^{1.84}$ (that is, where the amount of data stored per capita is proportional to the amount of energy consumed per capita).

The power available for computing scaling as $t^{0.84}$ while the energy consumed per operation $\sim t^{-1}$, the processing rate or computing power scales as $t^{1.84}$. That is, all knowledge ($t^{1.84}$) can be accessed in a fixed length of time ($\sim t^{1.84-1.84}$), say one lifetime, at all epochs.

5.4 Effect of Background Radiation

Hitherto we have considered the background radiation only as the limiting thermodynamic cold heat sink; since according to the Second Law of Thermodynamics it is apparently useless for doing work. The 'heat death of the universe' is supposed to occur when all energy sources reach the temperature of the background, when their entropy is maximised and no more work can be done. However, since the universe continues to expand, and the temperature to fall, entropy can increase without limit, the heat-death never occur, and in principle even the maximal-entropy background radiation be used again.

If black holes have mass $\sim t^r$, $r \geq 1$ they will become cooler than their surroundings and absorb the background radiation. In these circumstances the effective value of the mass fraction a may equal (or exceed) unity, allowing $P_T \sim t$ and the ratio of black hole mass to background radiation mass to be held constant. Such a scheme may be useful in rehabilitating regions where all matter decays before mankind can reach them; this would appear to solve the problem of Section 4.1.

6. CONCLUSIONS

In this paper we have argued that widely held beliefs concerning the necessity for long-term zero population growth and the finitude of natural resources are in error. We have reached the remarkable conclusion that mankind can grow in population, knowledge and power forever.

It must be emphasised that these scenarios are not to be taken as actual predictions. Many of the details presented here would have been inconceivable scarcely a decade ago and the growth of science and technology is sure to outdate them within centuries (let alone 10^{100} years!). Nevertheless, since new knowledge can but increase our capabilities, we shall be able to do at least as much as I have described. This paper is thus a form of "existence proof" of an unlimited, an ever-open, future.

Does the Second Law of Thermodynamics really imply that we live in a dying universe? Many cosmologists of this century have accepted this depressing interpretation: "The more the universe seems comprehensible, the more it also seems pointless" [32]. Is human civilisation really doomed? Many think so: "It seems to me, then, that by 2000AD...man's social structure will have utterly collapsed...Nor is there likely to be a chance of recovery thereafter" [33].

But now, rejecting both sad claims, we may return to the hope of illimitable progress that is held at the heart of Western civilisation in the Judeo-Christian tradition [34, 35], in which, despite the ephemeral nature of this world, the ultimate hegemony of entropy is explicitly rejected: "And I saw a new heaven and a new earth...'Behold, I make all things new'" [36]; and a doctrine of eternal growth proclaimed: "And of the increase of his government and of peace there shall be no end" [37].

If then, population can continue to grow, without ecological burdens or economic burdens (as argued in Ref. 7) and without fear of "Malthusian catastrophe" (as argued in this paper), we ought still to ask whether such growth is desirable – for, as we have already seen, value judgements are essential to a complete study of this problem. Personal bias in favour of small populations may perhaps be suspected where a "global village" of relatively low population is recommended as the humanly optimum solution [38], but it is apparent that such arguments are usually based less on supposed intrinsic merits than upon Malthusian doctrines.

Is population growth, then, a good thing? I submit that, insofar as parents obtain pleasure from their children and have a desire for large families, it is good that their desires should be fulfilled. For it is pleasant to be part of a growing society, enlivened by children and young people. For this we must have population growth. Furthermore, I submit that, insofar as a person's life has on balance a positive value (and most people believe their own lives to have such value), is it good that many lives should be lived. For the more people there are, the greater the value, and the greater the total good that they can engender. For these – and other – reasons I would claim that population growth is indeed intrinsically a good thing, a conclusion which is in accord both with the utilitarian criterion of the "greatest good of the greatest number" and the biblical injunction to "be fruitful and multiply" [39].

We conclude therefore that it is both possible and desirable for mankind to grow in numbers, knowledge and power indefinitely.

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